CRITICISM FOR RIEMANN'S THEOREM ON A SUM OF CONDITIONALY CONVERGENT SERIES

In the beginning we shall prove that the theorem is incorrect. So, the theorem runs that by permutation terms of a conditionally converging series, the sum of a series can be reduced in anyone to the beforehand given number. Thus in a proof a sum of a series reduce in this number by successive approximations, in turn summarizing some of positive and negative terms. For an alternating in sign of a series $\mathsf{q}_{2n} > 0, \mathsf{q}_{2n-1} < 0$, having up to permutation a sum S, reduced to number $L > S$, the positive terms are summarized until their sum will exceed L :

$$\mathsf{q}_2 + \mathsf{q}_4 + \cdots + \mathsf{q}_{2m} > L$$

Then the negative terms increase to an available sum until the sum will less than L :

$$\mathsf{q}_2 + \mathsf{q}_4 + \cdots + \mathsf{q}_{2m} + \mathsf{q}_1 + \mathsf{q}_3 + \cdots + \mathsf{q}_{2k-1} < L$$

After each operation of successive approximations the partial sum of a series will be distinct from L on magnitude smaller absolute values last addend of a term.

Let's remark at first that the purpose of this permutation is that m was more than k. And secondly that in a proof a limit of partial sums, as a sum obtained after permutation, is declared but in an explicit aspect is not written. Let's make that has overlooked to make the Riemann, namely - write the limits:

$$\lim_{k\to\infty} \sum_{n=1}^{k} \mathsf{q}_n = \lim_{k\to\infty} \left(\sum_{n=1}^{k} \mathsf{q}_{2n} + \sum_{n=1}^{k} \mathsf{q}_{2n-1} \right) = S$$

$$\lim_{k,m\to\infty} \left(\sum_{n=1}^{k} \mathsf{q}_{2n} + \sum_{n=1}^{k} \mathsf{q}_{2n-1} + \sum_{n=k+1}^{m} \mathsf{q}_{2n} \right) = L$$

$$\lim_{k,m \to \infty} \left(\sum_{n=k+1}^{m} q_{2n} \right) = L - S$$

$$q_n \to 0 \Rightarrow (m - k) \to \infty$$

Thus that at any, as much as small difference the given number from a sum of a series, difference of amounts summable even and odd terms of a series aspires to infinity, makes a proof incorrect.

To prove an invariance of a sum conditionally converging alternating in sign of a series at permutations of its terms it is possible by two modes:

At first there is obvious that if alternating in sign the series q_n has a sum S, then $\forall\forall m, k$ if

$$\sum_{n=1}^{m} q_{2n} + \sum_{n=1}^{k} q_{2n-1} = L$$

that

$$\sum_{n=m+1}^{\infty} q_{2n} + \sum_{n=k+1}^{\infty} q_{2n-1} = S - L$$

Secondly for any odd term of a series, worth at any permutation on any place, there is an even term with the next index and series, formed from their sums $\Sigma_{n=1}^{\infty}(q_{2n-1} + q_{2n})$ exists and absolutely converges, and therefore satisfies to a condition of the Dirichlet's theorem about an invariance of a sum at permutation.

The invariance of a sum of any alternating series is in the same way proved.

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